UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/22 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question Number	Mark Scheme Details			Part Mark	Total
1	*EITHER:* Use $C = I\alpha$ to find angular acceleration α: (omitting minus sign loses this A1 only)	$\alpha = -48 \times 0{\cdot}3 / 18$ [$= -0{\cdot}8$]	M1 A1		
	Integrate or use $\omega_1 = \omega_0 + \alpha t$ to find time t:	$t = (2 - 6)/\alpha = 5$ [s]	M1 A1		
	OR: Use energy to find angle θ:	$\theta = \frac{1}{2}\, 18(6^2 - 2^2)/(48 \times 0{\cdot}3)$ [$= 20$]	(M1 A1)		
	Use $\theta = \frac{1}{2}(\omega_0 + \omega_1)t$ to find time t:	$t = \theta / \frac{1}{2}(6 + 2) = 5$ [s]	(M1 A1)	4	**[4]**
2	Find two momentum eqns, e.g.:	$3mv_{3m} = 3m \times 2u - 4mu$ *or* $mv_m = -mu + 4mu$ *or* $3mv_{3m} + mv_m = 3m \times 2u - mu$	M1 M1		
	Solve to find both speeds after colln.:	$v_{3m} = 2u/3$ *and* $v_m = 3u$	A1		
	Find total loss in KE:	$\frac{1}{2}m\,\{3(2u)^2 + u^2 - 3v_{3m}^2 - v_m^2\}$	M1 A1		
		$= \frac{1}{2}m\,(12 + 1 - 4/3 - 9)\,u^2$ $= (13/2 - 31/6$ *or* $16/3 - 4)\,mu^2$ $= (4/3)\,mu^2$ **A.G.**	A1	6	**[6]**
3	Use conservation of energy:	$\frac{1}{2}mv^2 = \frac{1}{2}m(7ga/2) - mga\,(1 - \cos\theta)$	B1		
		[$v^2 = 3ga/2 + 2ga\cos\theta$]			
	Equate radial forces:	$R = mv^2/a + mg\cos\theta$	B1		
	Eliminate v to find R:	$R = (3mg/2)\,(1 + 2\cos\theta)$ **A.G.**	M1 A1	4	
(i)	Find $\cos\theta_1$ when $R = 0$:	$\cos\theta_1 = -\frac{1}{2}$ [$\theta_1 = 2\pi/3$]	B1		
	Find speed at this point:	$v_1^2 = \frac{1}{2}\,ga$, $v_1 = \sqrt{(\frac{1}{2}\,ga)}$	M1 A1	3	
(iii)	*EITHER:* Use energy to find reqd speed v_2:	$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - mga\cos\theta_1$ *or* $\frac{1}{2}m(7ga/2) - mga$	M1		
	Simplify:	$\frac{1}{2}v_2^2 = \frac{1}{4}ga + \frac{1}{2}ga$, $v_2 = \sqrt{(3ga/2)}$	M1 A1	3	
	OR: Find horiz. comp. of v_2:	$v_1\cos(\pi - \theta)$ [$= \frac{1}{2}v_1 = \sqrt{(ga/8)}$]	(M1)		
	Find vertical comp. of v_2:	$\sqrt{\{v_1^2 \sin^2(\pi - \theta) + ga\}}$ [$= \sqrt{(11ga/8)}$]	(M1)		
	Combine comps. to find v_2:	$v_2 = \sqrt{(ga/8 + 11ga/8)} = \sqrt{(3ga/2)}$	(A1)		**[10]**

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Question Number	Mark Scheme Details			Part Mark	Total
4	Find MI of disc D about O:	$I_D = ½(9m)(3a)^2 = (81/2)ma^2$	B1		
	Find MI of removed disc D' about O:	$I_{D'} = ½\,ma^2$	B1		
	Find MI of circular lamina about O:	$I_O = I_D - I_{D'}$ $[= 40\,ma^2]$	M1		
	Find MI of circular lamina about A:	$I_A = I_O + 8m(3a)^2$ *or* $½(243 - 19)ma^2$ $= 112ma^2$ **A.G.**	M1 A1	5	
	Find MI of lamina plus particle about A:	$I_A' = I_A + (3m)(6a)^2 = 220ma^2$	B1		
	State or imply that speed is max when AB vertical		M1		
	Use energy when AB vertical (or at general point):	$½\,I_A'\omega^2 = 8mg \times 6a + 3mg \times 12a$ *or* $2 \times 11mg \times 42a/11$ $[= 84mga]$	M1 A1		
	Substitute for I_A' and find max ang. speed ω (or ω^2):	$\omega^2 = 84mga/110ma^2 = 42g/55a$	A1		
	Equate $6a\omega$ to $k\sqrt{(ga)}$ to find k:	$k = \sqrt{(36 \times 42/55)} = 5{\cdot}24$	M1 A1	7	**[12]**
5	Resolve forces vertically:	$R_C + F_A = 3mg + 5mg$	B1		
	Take moments about B for AB:	$3aF_A = (3a/2)3mg$ $[F_A = 3mg/2]$	M1 A1		
	Combine to find R_C:	$R_C = 8mg - 3mg/2 = 13mg/2$ **A.G.**	M1 A1	5	
	Substitute for R_C in above resln. eqn to find F_A:	$F_A = 8mg - 13mg/2 = 3mg/2$	B1		
	Take moments about B for BC:	$4aF_C = 3aR_C - (3a/2)5mg$	M1		
	Substitute for R_C to find F_C:	$F_C = 3mg$	A1		
	Find limiting value μ_C for μ at C [or A] (A.E.F.) :	$\mu_C = 6/13$ $[= 0{\cdot}462$ *or* $\mu_A = 0{\cdot}5]$	M1 A1		
	Relate R_A, F_C by e.g. horizontal resolution:	$R_A = F_C$ $[= 3mg]$	B1		
	Deduce least possible value of μ for system:	$\mu_{min} = \max[\mu_A, \mu_C] = 0{\cdot}5$	B1	7	**[12]**
6	Find prob. that 10^{th} bulb is first defective one:	$(1 - 0{\cdot}01)^9 \times 0{\cdot}01 = 0{\cdot}00914$ (allow $0{\cdot}00913$)	M1 A1	2	
	State or find $E(N)$:	$E(N) = 1/0{\cdot}01 = 100$	B1	1	
	Formulate condition for n: (equality throughout loses this M1 only)	$P(N \le n) = 1 - P(N > n)$ $= 1 - 0{\cdot}99^n > 0{\cdot}9$, $0{\cdot}99^n < 0{\cdot}1$	M1		
	Take logs (any base) to give inequality for n:	$n > \log 0{\cdot}1 / \log 0{\cdot}99$	M1		
	Find n_{min}:	$n > 229{\cdot}1$, $n_{min} = 230$	A1	3	**[6]**

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7	Consider differences e.g. outdoor – indoor times:	0·1 2·1 -0·1 0·2 2·4 0·5 2·8 –2·6	M1		
	Calculate sample mean:	$\bar{d} = 5{\cdot}4 / 8 = 0{\cdot}675$	M1		
	Estimate population variance: (allow biased here: 2·679 *or* $1{\cdot}637^2$)	$s^2 = (25{\cdot}08 - 5{\cdot}4^2/8) / 7$ $= 3{\cdot}062$ *or* $1{\cdot}750^2$	M1		
	State hypotheses (A.E.F.):	H_0: $\mu_o - \mu_i = 0$, H_1: $\mu_o - \mu_i \neq 0$	B1		
	Calculate value of t (to 3 sf):	$t = \bar{d}/(s/\sqrt{8}) = 1{\cdot}09$	M1 *A1		
	State or use correct tabular t value: (or can compare $\bar{d}$ with 1·46[3])	$t_{7,\,0.975} = 2{\cdot}36[5]$	*B1		
	Correct conclusion (AEF, dep *A1, *B1):	No difference between mean times	B1	8	**[8]**
8	Relate F(x) to Poisson distribution (ignore x < 0):	$F(x) = 1 - P(X > x)$ $= 1 - P(\text{no flaws in length } x)$	M1		
		$= 1 - e^{-(x/100)1{\cdot}6} = 1 - e^{-0{\cdot}016x}$ **A.G.**	M1 A1		
	Equate F(x) to $1 - e^{-\lambda x}$ or f(x) to $e^{-\lambda x}$ to find mean λ:	$\lambda = 1/0{\cdot}016 = 62{\cdot}5$	B1	4	
(i)	Formulate eqn for median m of X:	$1 - e^{-0{\cdot}016m} = ½$	M1		
	Find value of m:	$m = -\ln ½ / 0{\cdot}016 = 43{\cdot}3$	M1 A1	3	
(ii)	Find $P(X \geq 50)$ (or > 50):	$1 - F(50) = e^{-0{\cdot}8} = 0{\cdot}449$	M1 A1	2	**[9]**
9	Calculate sample mean:	$\bar{d} = 42{\cdot}5 / 8 = 5{\cdot}3125$	M1		
	Estimate population variance: (allow biased here: 1·940 *or* $1{\cdot}393^2$)	$s^2 = 15{\cdot}519 / 7$ $= 2{\cdot}217$ *or* $1{\cdot}489^2$	M1		
	State hypotheses (A.E.F.):	H_0: $\mu = 4{\cdot}5$, H_1: $\mu > 4{\cdot}5$	B1		
	Calculate value of t (to 3 sf):	$t = (\bar{d} - 4{\cdot}5)/(s/\sqrt{8}) = 1{\cdot}54$	M1 *A1		
	State or use correct tabular t value: (or can compare $\bar{d}$ with 4·5 + 0·998 = 5·49[8])	$t_{7,\,0.95} = 1{\cdot}89[5]$	*B1		
	Correct conclusion (AEF, dep *A1, *B1):	Mean is not greater than 4·5	B1	7	
	Find confidence interval (allow z in place of t) e.g.:	$5{\cdot}3125 \pm t\sqrt{\{15{\cdot}519/(7 \times 8)\}}$	M1		
	Use of correct tabular value:	$t_{7,\,0.975} = 2{\cdot}36[5]$	A1		
	Evaluate C.I. correct to 3 s.f.:	$5{\cdot}31 \pm 1{\cdot}24[5]$ *or* [4·07, 6·56]	A1	3	**[10]**

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Question Number	Mark Scheme Details			Part Mark	Total
10	Find expected values (to 1 d.p.): (lose A1 if rounded to integers)	54·72 52·992 36·288 40·28 39·008 26·712	M1 A1		
	State (at least) null hypothesis (A.E.F.):	H_0: Preferences are independent	B1		
	Calculate value of χ^2:	$\chi^2 = 0{\cdot}5095 + 0{\cdot}0186 + 0{\cdot}5067$ $+ 0{\cdot}6921 + 0{\cdot}0252 + 0{\cdot}6883$ $= 2{\cdot}44$ or $2{\cdot}45$	M1 A1		
	State or use correct tabular χ^2 value (to 3 sf):	$\chi_{2,\,0.95}{}^2 = 5{\cdot}99[1]$	B1		
	Conclusion consistent with values (A.E.F):	Preferences are independent	A1 √	7	
	Calculate new value $\chi_{\text{new}}{}^2$ of χ^2:	$\chi_{\text{new}}{}^2 = N \times \chi^2$	M1		
	State or use correct tabular χ^2 value:	$\chi_{2,\,0.99}{}^2 = 9{\cdot}21$	B1		
	Find $N_{\min}$:	$N > 9{\cdot}21/2{\cdot}45$, $N_{\min} = 4$	M1 A1	4	**[11]**
11 (a)	Resolve vertically at equilibrium with extn. e:	$4mge / l = mg$ $[e = \frac{1}{4}l]$	B1		
	Use Newton's Law at general point:	$m\,\mathrm{d}^2x/\mathrm{d}t^2 = mg - 4mg(e+x)/l$ $[\,or\ -mg + 4mg(e-x)/l\,]$	M1		
	Simplify to give standard SHM eqn: S.R.: Stating this without derivation (max 3/4):	$\mathrm{d}^2x/\mathrm{d}t^2 = -(4g/l)\,x$	A1 (B1)		
	Find period T using SHM with $\omega = \sqrt{(4g/l)}$:	$T = 2\pi/\sqrt{(4g/l)} = \pi\sqrt{(l/g)}$ **A.G.**	B1	4	
	Find speed v_E at E using $v^2 = \omega^2(A^2 - x^2)$ with $x = 0$:	$v_E = \omega l/8 = \frac{1}{4}\sqrt{(gl)}$	M1 A1		
	Find speed v_P before striking plane (A.E.F.):	$v_P = \sqrt{(gl/16 + 14gl/16)} = \frac{1}{4}\sqrt{(15gl)}$	M1 A1		
	Find comps. of speed V after striking plane:	Parallel to plane: $v_P \sin 30°$ or $\frac{1}{2} v_P$ or $\sqrt{(15gl/64)}$	B1		
		Normal to plane: $\frac{1}{3} v_P \cos 30°$ or $\frac{1}{3}(\sqrt{3}/2)\, v_P$ or $\sqrt{(5gl/64)}$	B1		
	Combine to find V:	$V^2 = 15gl/64 + 5gl/64 = 5gl/16$	M1		
		$V = \frac{1}{4}\sqrt{(5gl)}$ **A.G.**	A1	8	**[12]**

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Question Number	Mark Scheme Details		Part Mark	Total
(b) (i)	Find correlation coefficient r:	$\Sigma x = 8{\cdot}5$, $\Sigma x^2 = 9{\cdot}67$, $\Sigma xy = 3{\cdot}955$, $\Sigma y = 3{\cdot}85$, $\Sigma y^2 = 2{\cdot}0775$ $r = (3{\cdot}955 - 8{\cdot}5 \times 3{\cdot}85/8) / \sqrt{\{(9{\cdot}67 - 8{\cdot}5^2/8)(2{\cdot}0775 - 3{\cdot}85^2/8)\}}$ M1 A1		
	(A0 if only 3 s.f. used)	$= -0{\cdot}1356 / \sqrt{(0{\cdot}6387 \times 0{\cdot}2247)}$ A1 $= -0{\cdot}1356 / (0{\cdot}7992 \times 0{\cdot}4740)$ [*or* $-0{\cdot}01695 / \sqrt{(0{\cdot}07984 \times 0{\cdot}02809)}$ $= -0{\cdot}01695 / (0{\cdot}2826 \times 0{\cdot}1676)$] $= -0{\cdot}358$ *A1	4	
(ii)	State both hypotheses:	H_0: $\rho = 0$, H_1: $\rho < 0$ B1		
	State or use correct tabular one-tail r value:	$r_{8,\,2.5\%} = 0{\cdot}707$ *B1		
	Valid method for reaching conclusion:	Accept H_0 if $\lvert r\rvert <$ tabular value M1		
	Correct conclusion (AEF, dep *A1, *B1):	There is no negative correlation A1		
	Valid comment, consistent with values	No effect of S on R (A.E.F.) B1√	5	
(iii)	Find critical tabular one-tail r value:	$r_{16,\,5\%} = 0{\cdot}426$ *or* $r_{15,\,5\%} = 0{\cdot}441$ M1 A1		
	Deduce range of possible values of N:	$N \geq 16$ A1	3	**[12]**

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